Economy Exercise

This section focuses on your ability to model, interpret, and calibrate economic parameters.

Part 1: Deriving Expected Value

Please answer the questions below using the following assumptions:

PvE and PvP Economic Data	
PvP Win Rate	50%
PvP Gold per Win	100
PvE Enemies Encountered per Attack	10
PvE Chance to Drop Gold	20%
PvE Gold per Drop	30

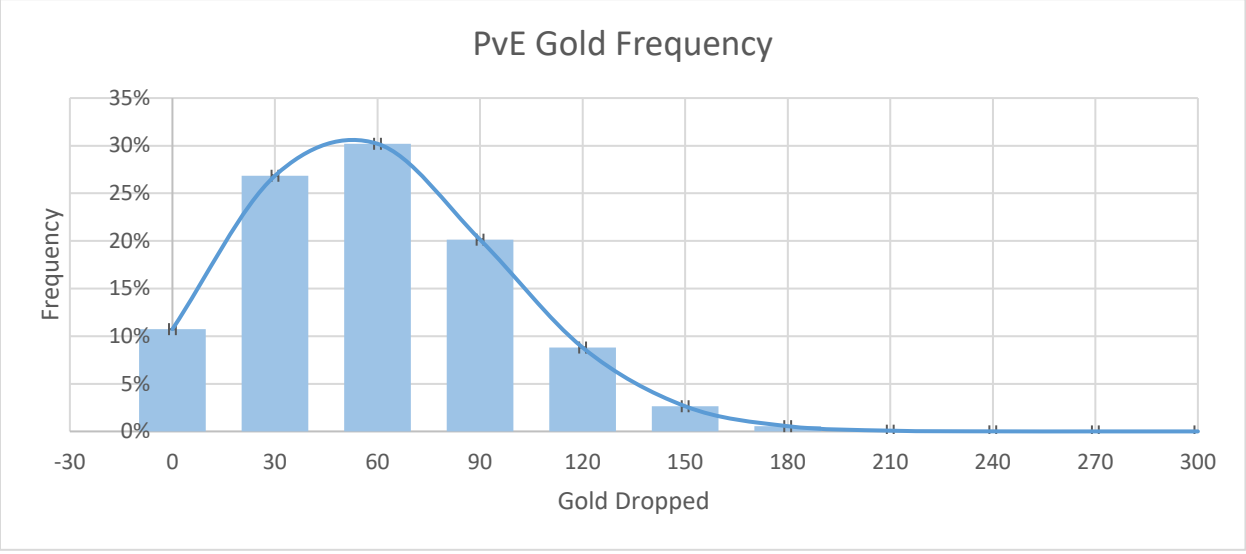
1. What is the expected value, denominated in gold, of PvP and PvE?

Expected value is 50 gold for PvP and 60 gold for PvE.

2. Provide frequency distributions based on PvP and PvE expected value.

PvE Gold Frequency		
Drops	**Gold**	**Freq %**
0	0	10.74%
1	30	26.84%
2	60	30.20%
3	90	20.13%
4	120	8.81%
5	150	2.64%
6	180	0.55%
7	210	0.08%
8	240	0.01%
9	270	0.00%
10	300	0.00%

PvP Gold Frequency		
Win	**Gold**	**Freq %**
0	0	50.00%
1	100	50.00%



3. **Provide summary statistics based on these results.**

PvE Summary Statistics (single encounter)				
	Drops		Gold	
Statistic	Low	High	Low	High
Mean	2		60	
Standard Deviation	1.26		37.95	
Variance	1.6		1440	
Expected Value	2		60	
1-Sigma	0.74	3.26	22.05	97.95
2-Sigma	-0.53	4.53	-15.89	135.89
1-Sigma (normalized)	1	3	30	90
2-Sigma (normalized)	0	5	0	150

- Most (77%) of PvE encounters will result in 1-3 drops (30-90 gold) with 2 drops (60 gold) most likely.
- There is an 11% chance that there will be 0 drops, and an 11% chance there will be 4-5 drops (120-150 gold.)
- There is less than 1% chance of 6 or more drops (180-300 gold.)

PvP Summary Statistics (single encounter)				
	Wins		Gold	
Statistic	Low	High	Low	High
Mean	0.5		50	
Standard Deviation	0.5		50	
Variance	0.25		2500	
Expected Value	0.5		50	
1-Sigma	0	1	0	100
2-Sigma	-0.5	1.5	-15	150
1-Sigma (normalized)	0	1	0	100
2-Sigma (normalized)	0	1	0	100

- Half (50%) of all PvP encounters will result in a win (100g) the other 50% will result in a loss (0 gold.)

4. **Assuming that a player engages in 5 combats per day with a hard limit of 3 per type of encounter, what is the expected total gold production per day? Assume gold-maximizing behavior.**

Expected total production per day should be 280 gold. (3 x PvE + 2 x PvE)

5. **Why might a player choose PvE over PvP or vice-versa? Provide both qualitative and quantitative interpretations.**

<u>Pro PvE:</u>

- Highest expected gold value (quantitative)
- Highest consistency in returns (quantitative)
- Favors players who underperform in PvP combat (qualitative and quantitative)
- Lower risk favors those who are risk averse (qualitative)
- Cooperative play (qualitative)

Pro PvP:

- Highly skilled players can “beat the odds” and get better than expected returns (quantitative)
- More engaging for aggressive players and risk seekers (qualitative)
- More challenging and uncertain – less boring (qualitative)
- Feels like bigger payoffs / gambling behaviors (qualitative)

Part 2: Extrapolating Costs

Assume there are 10 tiers of an item in the game and you have been given some rough targets based on days needed to acquire the gold required to upgrade to each tier.

Progression Targets	
Tier	**Days**
1	0.5
4	3
10	20

1. **Using the gold production per day from the previous exercise, extrapolate a cost curve based on these targets.**

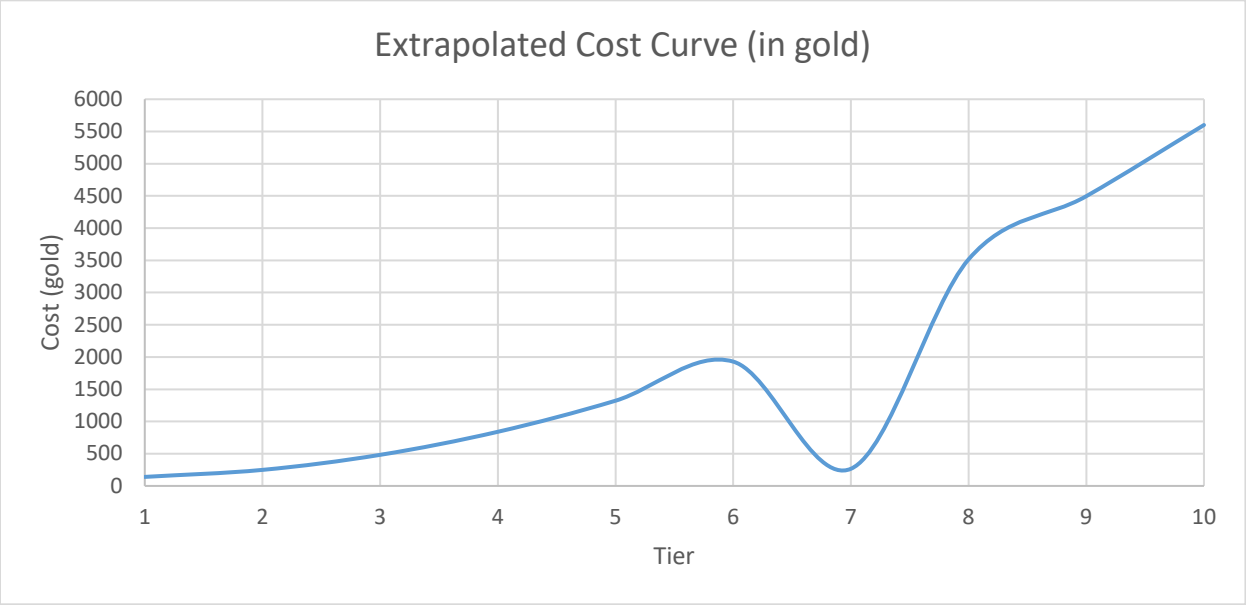
Extrapolated Cost Curve			
Tier	**Days (d)**	**Cost (g)**	**delta-d**
1	**0.5**	140	0.50
2	0.9	249	0.39
3	1.7	482	0.83
4	**3.0**	840	1.28
5	4.7	1322	1.72
6	6.9	1929	2.17
7	9.5	2660	2.61
8	12.6	3516	3.06
9	16.1	4496	3.50
10	**20.0**	5600	3.94

Extrapolated curve based on the following quadratic equation:

$$y = \frac{2}{9}x^2 - \frac{5}{18}x + \frac{5}{9}$$

x = Tier
y = Days

Note: The delta-d value reflects the difference in days from tier to tier and represents the slope of the cost curve at each tier.



2. **Based on your extrapolated curve, provide summary statistics that you feel are valuable.**

Summary Stats on Cost Curve (per tier)			
Statistic	**Days**	**Cost**	**delta-d**
Mean	7.6	2123.3	2.0
Median	5.8	1625.6	1.9

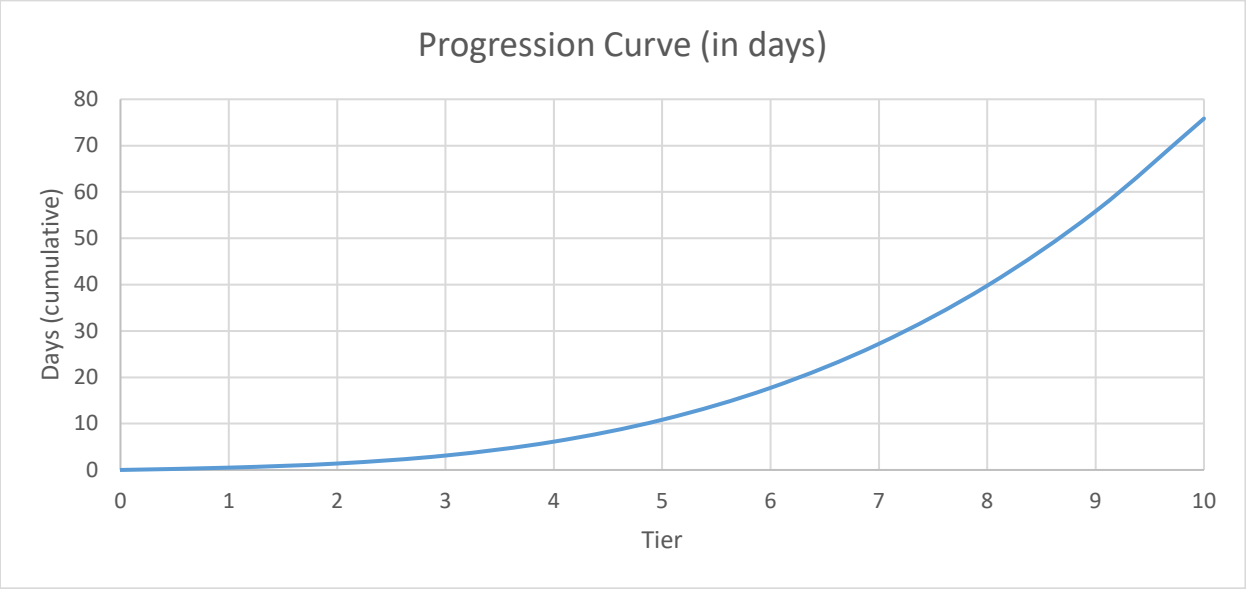
Summary Stats on Cost Curve (per quartile)					
				Cost	
Statistic	**Mean d/tier**	**Mean g/tier**	**Tiers**	**Min**	**Max**
1st Quartile	1.0	290.4	1-3	140.0	571.7
2nd Quartile	3.9	1081.1	4-5	572.7	1625.6
3rd Quartile	8.2	2294.4	6-7	1626.6	3301.7
4th Quartile	16.2	4537.0	8-10	3302.7	5600.0

Note: Quartiles are based a N-1 approach (including median), using the per-tier cost values. Tier estimates are best fit.

Progression Data			
Tier	**cum-d**	**cum-g**	**prog %**
0	0	0	0
1	0.5	140	0.7%
2	1.4	389	1.8%
3	3.1	871	4.1%
4	6.1	1711	8.1%
5	10.8	3033	14.3%
6	17.7	4962	23.4%
7	27.2	7622	35.9%
8	39.8	11138	52.5%
9	55.8	15633	73.6%
10	75.8	21233	100.0%

Summary Stats on Progression						
Statistic	**Mean d/tier**	**Mean g/tier**	**Tiers**	**Total Days**	**Total Cost**	**Prog %**
25% Progress	3.0	827.0	1-6	17.7	4962	23.4%
50% Progress	11.0	3087.8	7-8	39.8	11138	52.5%
75% Progress	16.1	4495.6	9	55.8	15633	73.6%
100% Progress	20.0	5600.0	10	75.8	21233	100.0%

Cumulative days = cum-d
Cumulative gold = cum-g
Total progress % = prog%



Some useful takeaways from this summary data:

- Progression from base item to fully upgraded tier 10 item will take roughly 76 days (21,233 gold) for top players.
  - On average it will take 7-8 days per tier for the top players to reach tier 10.
- The time it takes to progress through the upgrade curve is pretty significant at the upper tiers.
  - It takes an average of 3 days per tier to get through the first 25% of the upgrades (tiers 1-6)
  - It takes an average of 11 days per tier to get through the second 25% of the upgrades (tiers 7-8)
  - It takes roughly 16 days to get through the third 25% of the upgrades (tier 9)
  - It takes roughly 20 days to get through the last 25% of the upgrades (tier 10)
  - It takes roughly the same amount of time to upgrade from tier 0 to 8 as it does to upgrade from 8 to 10.

3. What are some of the advantages and disadvantages associated with the curve you have extrapolated?

Advantages:

- Cost curve is predictable.
- Cost curve is forgiving at the lower tiers, allowing players to make good progress early on to improve engagement.
- The data allows us to best determine the impact of each upgrade tier to ensure that all upgrades are balanced against the cumulative cost required to reach their tier. This helps overall balance.
  - For example: By tier 6, the players have spent roughly 25% of the total gold cost of the fully upgraded item, so the impact of the upgrades to that point should be roughly 25% of the impact of the fully upgraded tier 10 item.

Disadvantages:

- Cost curve seems arbitrary rather than being tied to measured player behaviors. Cost curves should be informed by metrics derived from known values for how much time players spend per day, etc.
- Curve may be too steep at the higher end, and could result in reduced player engagement (burnout). Particularly if they are trying to upgrade multiple items at the same time. We don't want to lose players, so adding metrics to help track this would be useful.
- In addition to the above, the steepness of the curve could potentially mean that upgrades at lower tiers may need to be statistically insignificant to ensure that the upgrades at higher tiers have a large enough impact.

4. From a strictly quantitative standpoint, how might you calibrate this curve to mitigate some of its disadvantages? What other features might you introduce to achieve a more optimal result?

I would calibrate the curve (adjusting delta-d) to revise the curve to best suit player behaviors and business goals. As an example of business goals, perhaps we'd have some goal in mind, like "10% of all players should reach max tier by the time the next new content patch is available" in order to make sure that top players always have a carrot to keep them engaged.

As for additional features to help insure more optimal results, I'd add metrics to track player behaviors, actual gold generation values, and play time at the least. I'd then use them to help inform how best to tune the costs and progression model to optimize those curves to provide the best experience for the largest (or highest paying) cohort of players (and to insure new content introductions are timed to coincide with old content exhaustion.)